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Potential Applications of Piston Generated Unsteady Expansion Waves

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An analysis of unsteady, centered expansion waves is performed, where the flow is generated by impulsively withdrawing a piston from a gas at constant speed. Primary interest is in stagnation quantities, such as pressure, temperature, and density, which have a surprising variation. They decrease in the early part of the expansion, have a minimum when the Mach number is unity, and then proceed in the supersonic region to increase to values well in excess of their respective values ahead of the expansion. If the piston's motion is then impulsively stopped, a strong shock wave is generated. For a period of time, the flowfield between the piston and shock wave is uniform and motionless. The gas in this region is characterized by a low density and a high temperature. Two possible applications are thus suggested. First, chemical kinetic rate studies can be performed in the hot gas between the piston and the shock wave. Second, condensation processes can be studied in the low temperature gas ahead of the shock wave, while evaporation of the newly formed clusters can be analyzed in the high-temperature gas behind the shock wave.

Nomenclature

a = speed of sound
h = enthalpy
L = piston travel distance
M = Mach number
p = pressure
R = dimensionless density, ρ/ρ_1
t = time
Δt = estimated test time
T = temperature
u = velocity
u_p = piston speed
u_s = shock speed
U = dimensionless velocity, u/a_1
x = distance
η = similarity variable, $x/a_1 t$
γ = specific heat ratio
ρ = density
τ = dimensionless test time, $a_1\Delta t/L$

Subscripts

0 = stagnation condition
1 = undisturbed gas
2 = uniform flow region between piston and trailing edge of expansion
3 = uniform flow region between piston and shock wave
e = trailing edge of expansion
m = vacuum limit

Introduction

IT is common practice to introduce unsteady, centered expansions by considering the impulsive motion of a piston inside a cylinder.[1] Along a particle path in the expansion, stagnation quantities are not constant, as shown by the relation for the stagnation enthalpy h_0

$$\frac{Dh_0}{Dt} = \frac{1}{\rho}\frac{\partial p}{\partial t} \tag{1}$$

where D( )/Dt is the substantial derivative, and other variables are defined in the Nomenclature.

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The genesis of this work stems from teaching an undergraduate gas dynamic course. The author had the students determine the variation of the usual stagnation, or total, quantities in the expansion. This variation proved interesting, and a subsequent search of the literature failed to unearth a comparable analysis, even though it is straightforward. It has not been investigated probably because of the frequent comment that unsteady, isentropic, one-dimensional flow has qualitatively identical properties to a steady, two-dimensional flow where stagnation conditions are constant. The search demonstrated that unsteady, centered expansions are invariably computed using the method of characteristics, with application of the solution to shock tube flow. Occasionally, the particle path is determined. Apparently, stagnation quantities are not evaluated.

The analysis in the next section utilizes a similarity solution, because it is simpler, in preference to the method of characteristics. Static and stagnation conditions, along with the Mach number, are dependent only on the similarity variable. For reasons to be explained, the stagnation temperature can be quite high at the trailing edge of the expansion. By impulsively stopping the piston, a shock wave is formed. The shock propagates into a uniform flow, which is between the expansion and the piston, and which is designated as region 2 in Fig. 1. The flow between the piston and shock wave, region 3 in Fig. 1, is both uniform and quiescent, until the shock interacts with the expansion. The reason for stopping the piston is to recover, in a useful manner, the high stagnation temperature in region 2. Region 3 is thus characterized by a high temperature and low pressure and density.

The analysis in the next section is suggestive of several applications that are discussed in the final section along with a summary. These applications do not require impulsively starting and stopping the piston. Impulsive motion is assumed in the analysis for its simplicity.

Analysis

Expansion Solution

A piston starts impulsively from rest and moves to the left, Fig. 1, with a constant negative speed u_p. This motion generates an isentropic, centered expansion in a perfect gas. The equations of continuity and momentum are

$$\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial x} + \rho\frac{\partial u}{\partial x} = 0 \tag{2a}$$

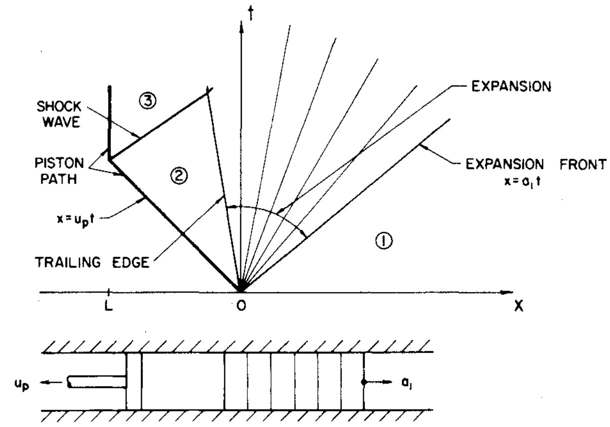

Fig. 1 *x-t* diagram and schematic of piston motion.

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+\frac{a^2}{\rho}\frac{\partial \rho}{\partial x}=0 \tag{2b}$$

where the speed of sound is given by

$$a^2=\gamma(p/\rho)=a_1^2(\rho/\rho_1)^{\gamma-1} \tag{2c}$$

The gas ahead of the expansion in region 1 provides constant conditions that are used for nondimensionalizing variables.

The well-known solution for the flow inside the expansion is needed in the subsequent analysis and thus is provided only in outline. Equations (2) are transformed by means of

$$\eta=x/(a_1t) \tag{3}$$

$$U(\eta)=u(x,t)/a_1 \tag{4a}$$

$$R(\eta)=\rho(x,t)/\rho_1 \tag{4b}$$

$$(U-\eta)\frac{dR}{d\eta}+R\frac{dU}{d\eta}=0 \tag{5a}$$

$$(U-\eta)\frac{dU}{d\eta}+R^{\gamma-2}\frac{dR}{d\eta}=0 \tag{5b}$$

Boundary conditions at the expansion front, where $\eta=1$, are

$$U(1)=0, \qquad R(1)=1 \tag{6}$$

Equations (5) and (6) are easily solved, by first eliminating the $(U-\eta)$ factor, to provide

$$U=-\frac{2}{\gamma+1}(1-\eta) \tag{7a}$$

$$R=\left(\frac{2}{\gamma+1}+\frac{\gamma-1}{\gamma+1}\eta\right)^{2/(\gamma-1)} \tag{7b}$$

The pressure, temperature, and Mach number are then given by

$$p/p_1=R^\gamma \tag{7c}$$

$$T/T_1=R^{\gamma-1} \tag{7d}$$

$$M=-\frac{u}{a}=-U\left[\frac{R}{(p/p_1)}\right]^{1/2}=\frac{1-\eta}{1+(\gamma-1)\eta/2} \tag{7e}$$

Equations (7) apply only inside the expansion, where u is negative; hence, the minus sign in the Mach number definition.

Conditions at the fan's trailing edge, denoted by a subscript e, apply throughout region 2. The value for η_e is obtained by equating the piston speed u_p with u_e, to yield

$$U_e=\frac{u_p}{a_1}=-\left(\frac{2}{\gamma+1}\right)(1-\eta_e)$$

or

$$\eta_e=1+\frac{\gamma+1}{2}\frac{u_p}{a_1} \tag{8}$$

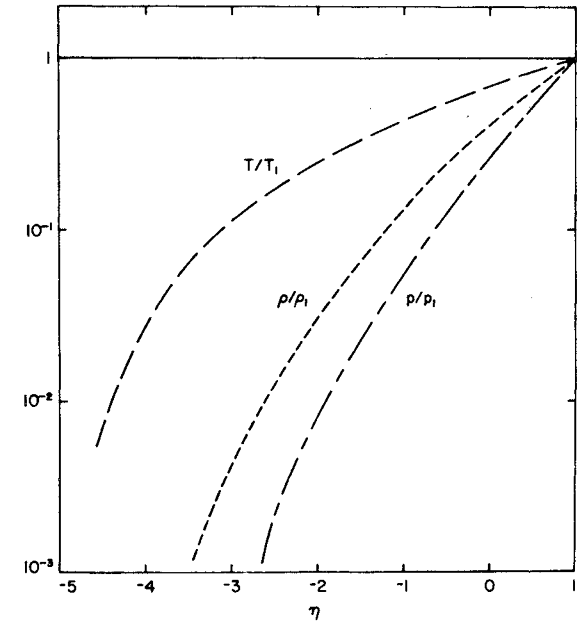

Fig. 2 Static pressure, temperature, and density in the expansion when $\gamma=1.4$.

Figure 2 shows the static pressure, temperature, and density when $\gamma=1.4$. By moving counterclockwise into the expansion, η decreases from unity. Equation (7e) shows that the Mach number is unity on the t axis, where $\eta=0$. If the expansion is sufficiently strong, the fan extends into the second quadrant of the x,t plane, where the flow is supersonic. A minimum η_{em} occurs for η_e when $p_e=\rho_e=T_e=0$ and $M_e=\infty$. From Eqs. (7e) and (8) this corresponds to

$$\eta_{em}=-2/(\gamma-1) \tag{9a}$$

$$u_p/a_1\leq -2/(\gamma-1) \tag{9b}$$

Cavitation occurs when the inequality sign in Eq. (9b) applies.

Stagnation Conditions

Within the fan, local stagnation conditions are readily obtained. For example, the temperature is

$$\frac{T_0(\eta)}{T_1}=\frac{T(\eta)}{T_1}\frac{T_0(\eta)}{T(\eta)}=\frac{2}{\gamma+1}+\frac{\gamma-1}{\gamma+1}\eta^2 \tag{10a}$$

where

$$\frac{T_0}{T}=1+\frac{\gamma-1}{2}M^2=\frac{\gamma+1}{2}\frac{1+(\gamma-1)\eta^2/2}{[1+(\gamma-1)\eta/2]^2}$$

The stagnation pressure and temperature are given by

$$\frac{p_0}{p_1}=\left[\frac{T_0(\eta)}{T_1}\right]^{\gamma/(\gamma-1)} \tag{10b}$$

$$\frac{\rho_0}{\rho_1}=\left[\frac{T_0(\eta)}{T_1}\right]^{1/(\gamma-1)} \tag{10c}$$

Static conditions in region 1 are also stagnation conditions, since $u_1=0$. Stagnation values for ρ, p, and T have a minimum when $\eta=0$, and are equal to their respective counterparts in region 1 on the $\eta=-1$ ray or characteristic. Figure 3 contains curves for these stagnation ratios and for the Mach number when $\gamma=1.4$. Stagnation values higher than their region 1 counterpart occur when $\eta_e<-1$. These high stagnation values can occur if $\gamma<3$, since $\eta_{em}\leq\eta_e$.

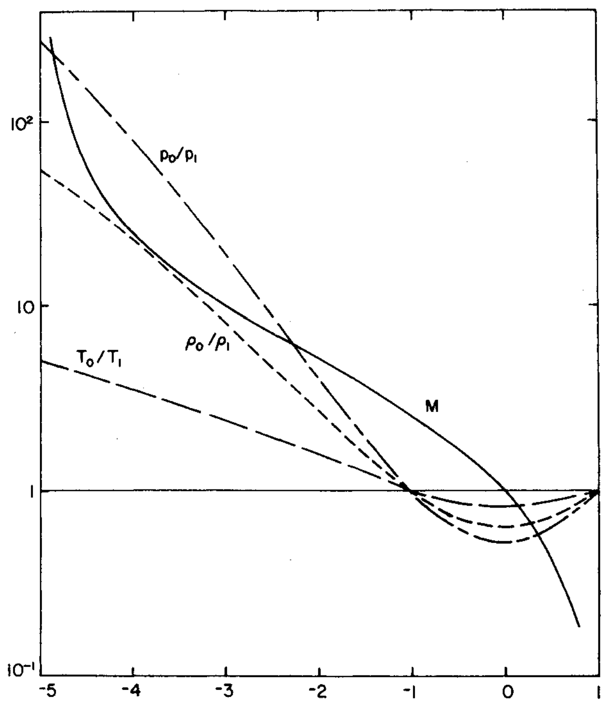

Fig. 3 Variation in the expansion of stagnation pressure, temperature, and density and the Mach number when $\gamma = 1.4$.

An explanation for the high stagnation values is evident from Eq. (1). In the first quadrant in Fig. 1, $(\partial p/\partial t)$ is negative at a fixed x, because the pressure falls inside the expansion as t increases. On the other hand, for a fixed, negative x (i.e., in the second quadrant), pressure increases with t in the expansion and $(\partial p/\partial t)$ is positive. Thus, $(\partial p/\partial t)$ changes sign at $x=0$. The pressure field therefore transfers energy from the subsonic region to the supersonic region. The transfer process is amplified by the low density in the supersonic region, as shown by the $(1/\rho)$ coefficient in Eq. (1).

There is no counterpart to the unsteady energy transfer process when a constant speed piston moves into a gas. In this case, the flow between the piston and shock wave is uniform.

Conditions Behind the Shock Wave

In region 3 flow conditions are found by using a coordinate system, denoted by a tilde, fixed with the shock wave. Static conditions ahead of and behind the shock are unaltered by this transformation.[2] Hence,

$$\tilde{T}_2 = T_2, \quad \tilde{T}_3 = T_3, \ldots \tag{11a}$$

and because $u_3 = 0$, and conditions in region 2 are the same as on η_e, we have

$$\tilde{T}_2 = T_2 = T_e, \quad \tilde{T}_3 = T_3 = T_{03}, \ldots \tag{11b}$$

The velocity transformation is given by

$$\tilde{u}_2 = u_2 - u_s, \quad \tilde{u}_3 = u_3 - u_s = -u_s \tag{12}$$

where u_s is the shock wave speed. Eliminating u_s and introducing the Mach numbers

$$u_2 = -a_2 M_e, \quad \tilde{u}_2 = -a_2 \tilde{M}_2, \quad \tilde{u}_3 = -a_3 \tilde{M}_3 \tag{13}$$

yields the relation

$$\tilde{M}_2 - (T_3/T_2)^{1/2} \tilde{M}_3 = M_e \tag{14}$$

By means of the normal shock equations

$$\left(\frac{T_3}{T_2}\right)^{1/2} = \left(\frac{2}{\gamma+1}\right) \frac{1}{\tilde{M}_2} [1 + (\gamma-1)\tilde{M}_2^2/2]^{1/2} \times [\gamma \tilde{M}_2^2 - (\gamma-1)/2]^{1/2} \tag{15a}$$

$$\tilde{M}_3 = \left[\frac{1+(\gamma-1)\tilde{M}_2^2/2}{\gamma\tilde{M}_2^2 - (\gamma-1)/2}\right]^{1/2} \tag{15b}$$

we obtain from Eq. (14)

$$\tilde{M}_2 = \frac{1}{2}\left\{\frac{\gamma+1}{2} M_e + \left[\left(\frac{\gamma+1}{2}\right)^2 M_e^2 + 4\right]^{1/2}\right\} \tag{16}$$

The trailing edge Mach number M_e is given by Eqs. (7) and (8) as

$$M_e = -\frac{(u_p/a_1)}{1+(\gamma-1)(u_p/a_1)/2} \tag{17}$$

With $\tilde{M}_2$ known, the usual normal shock equations, such as Eqs. (15), are used to find conditions in region 3. The shock speed u_s is then given by

$$u_s/a_1 = U_e + (T_e/T_1)^{1/2} \tilde{M}_2 \tag{18}$$

A useful parameter is the time Δt between shock wave initiation, and when the shock encounters the trailing edge of the expansion. During this time interval the gas in region 3 is quiescent and uniform, and consequently experimentally useful. A simple calculation shows this interval is given by

$$\tau = \frac{a_1}{L}\Delta t = \frac{1}{1-\eta_e}\left(\frac{1+\tilde{M}_2}{M_e} - \frac{\gamma+1}{2}\right) \tag{19}$$

Figure 4 shows the temperature, pressure, and density in region 3, along with τ, when $\gamma = 1.4$. Equation (9a) yields $\eta_{em} = -5$ for this γ. It is easy to show that in the limit

$$\eta_e \to \eta_{em} = -[2/(\gamma-1)] \tag{20a}$$

we obtain

$$\frac{T_{0e}}{T_1} = \frac{2}{\gamma-1}, \quad \frac{p_{0e}}{p_1} = \left(\frac{2}{\gamma-1}\right)^{\gamma/(\gamma-1)}, \quad \frac{\rho_{0e}}{\rho_1} = \left(\frac{2}{\gamma-1}\right)^{1/(\gamma-1)} \tag{20b}$$

$$\frac{T_3}{T_1} = \frac{2\gamma}{\gamma-1}, \quad \frac{p_3}{p_1} = 0, \quad \frac{\rho_3}{\rho_1} = 0 \tag{20c}$$

$$\frac{u_s}{a_1} = 1, \quad \tau = 0 \tag{20d}$$

Whereas p_3 and ρ_3 go to zero, Fig. 4, T_3 has a finite value. The interval τ is zero because the trailing edge of the expansion is coincident with the piston's surface. Because τ and ρ_3 are zero, this limit is not experimentally useful. A somewhat larger η_e value, say -4, however, might prove viable. For this η_e, we have

$$T_3/T_1 = 4.90, \quad p_3/p_1 = 3.76 \times 10^{-3}, \quad \rho_3/\rho_1 = 7.67 \times 10^{-4} \tag{21}$$

and $\tau = 8.27 \times 10^{-3}$. The gas is thus hot and of low density in region 3. If L is 5 m, and the speed of sound a_1 is 340 m/s, corresponding to N_2 at 300 K, then Δt is 1.2 ms. From Eq. (8) a piston speed of 1.4 km/s is computed. Test time can be substantially increased and the piston speed substantially decreased by using a high molecular weight gas (such as SF_6) to decrease a_1.

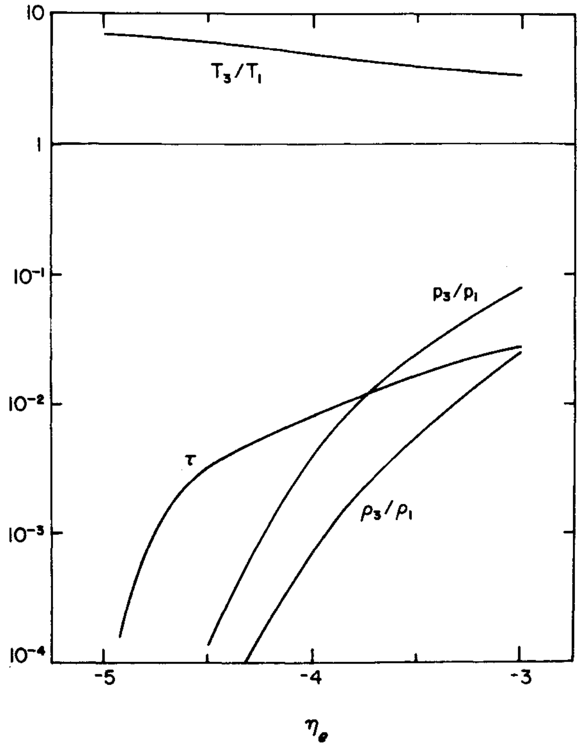

Fig. 4 Pressure, temperature, and density in region 3 and the test time when $\gamma = 1.4$.

Figure 4 illustrates the tradeoff between obtaining a high temperature and the need for finite values for τ and ρ_3. Fortunately, the T_3 curve falls slowly with η_e, while τ and ρ_3 increase rapidly. Equation (20c) shows a strong dependence of T_3 on γ. Gases with specific heat ratios smaller than 1.4 produce an appreciable higher region 3 temperature. In this regard a high vapor pressure gas is desirable to avoid condensation in region 2. (Alternatively, a region 1 temperature higher than room temperature should be considered.) A molecule that satisfies these requirements is SF_6.

Equation (11b) shows that $T_{03} = T_3$. Equations (20b) and (20c) however yield $T_{03} > T_{0e}$. In an unsteady flow, stagnation temperature is not constant across a shock wave.[2] In a coordinate system fixed with the shock wave T_{03} does equal $\tilde{T}_{0e}$.

Summary and Discussion

Unsteady, centered expansion waves produced by the impulsive motion of a piston are analyzed. Stagnation conditions in the expansion have a minimum on the characteristic where the Mach number is unity, and can attain significant values on the trailing-edge characteristic. These stagnation values are greater than their counterparts in the undisturbed gas when the similarity variable η_e is less than -1. As η_e approaches its vacuum limit, $-2/(\gamma-1)$, stagnation values on the trailing edge become much larger than their undisturbed counterparts.

The high value for the stagnation temperature can be realized experimentally by stopping the piston's motion. This creates a strong shock wave that reduces the stagnation pressure and density to low values, but further increases the stagnation temperature. The gas between the stopped piston and shock wave is quiescent, uniform, of high temperature, and of low density. The temperature in this region is sensitive to γ, with the temperature increasing inversely with $(\gamma-1)$ decreasing. Uniform flow terminates when the shock encounters the trailing edge of the expansion. While the temperature in the quiescent region is a maximum when $\eta_e = -2/(\gamma-1)$, this limiting condition is not experimentally useful, because the density and test time are zero. A value for η_e somewhat higher than this limit overcomes this difficulty.

Chemical kinetic rate studies could be performed in the hot, low density gas between the stopped piston and the shock wave. The rate processes are triggered by the strong shock wave with the reacting species typically being in low concentration relative to an inert diluent. Flush mounted windows on the cylinder wall near where the piston stops could be used for spectroscopic, or other optical, diagnostics. Other instrumentation, as long as they are flush with the cylinder wall, can be utilized. Kinetic rate studies are often done using a shock tube. Whether the piston-in-a-tube technique is competitive or superior to a shock tube needs to be evaluated. Such an evaluation, which is beyond the scope of this paper, would have to consider flexibility, test time, thermodynamic conditions, etc.

Condensation studies constitute another possible application. The rapid expansion creates a low static temperature in the gas between the moving piston and the expansion's trailing edge. A vapor mixed with a noncondensing carrier gas will condense in this region to form small clusters. (A carrier gas is not a necessary requirement; its use is optional.) The clusters then evaporate after they enter the shocked gas region. The piston-in-a-tube technique thus offers the unique opportunity to study in a single experiment both clustering and the subsequent evaporation of the clusters. Glass and co-workers in a series of excellent papers[3-5] have investigated water condensation using the expansion wave created in a shock tube. Once again, we see that the two techniques are competitive. In this case, however, the evaporative aspect cannot be done with a shock tube in a single experiment. (In a shock tube, the equivalent of the piston is the contact surface, which cannot be stopped.) The single experiment qualification is an important one. By systematically varying piston speed and thermodynamic conditions in the undisturbed gas, the size and number density of the clusters can be altered. Evaporation rate data, as a function of cluster size, should prove of value for determining bonding energy and the detailed mechanisms of cluster formation and evaporation.

The previous N_2 estimates, Eq. (21), are based on $\eta_e = -4$, which is appropriate only for kinetic studies. A suitable value for condensation studies would be $\eta_e = -1$, which yields

$$(T_e/T_1) = 0.444, \quad (T_3/T_1) = 1.36$$
$$u_p = -0.567 \text{ km/s}, \quad \Delta t = 4 \text{ ms}$$

With $T_1 = 300$ K, we have $T_2 = 132$ K and $T_3 = 408$ K, which are useful temperatures for condensation/evaporation experiments. Furthermore, the piston's speed is sharply reduced and the test time is increased. For similar temperatures Ar diluent provides an even greater reduction in piston speed (0.3 km/s) and an increase in test time (12 ms). The technique thus strongly favors condensation studies.

Acknowledgments

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